Cosmic Magnetic Fields Exert Negative Pressure and Act as a Cosmological Constant

Max Chaves
Escuela de Fisica, Universidad de Costa Rica
San Jose, Costa Rica
mchaves@cariari.ucr.ac.cr

May 18, 1999

Abstract

Recently there has been mounting evidence for the existence of a cosmological constant (CC). Here we show that cosmic magnetic fields act as a CC, having negative pressure and tending to cause cosmological acceleration. However, if present-day estimates of the intensity of these fields are correct, they would be too weak to be the cause of the acceleration. It is pointed out that *large-scale coherent* Yang-Mills field structures can also act as a CC.

For a few years now empirical evidence from different astrophysical sources has been accumulating that suggests that either the Universe is open or there is a nonzero cosmological constant (CC). Recently the issue has been greatly clarified by groups studying type Ia supernovae[1–4]. There is strong evidence that the cosmic expansion is accelerating and that there is a positive CC. Here we point out that cosmic magnetic fields act as a CC and are responsible for a part of the experimentally observed acceleration. This is surprising since there is a tendency to think that something that can act as a CC would have to be a very exotic object, but it is actually straightforward to show from the diagonal space components of the stress tensor of cosmic magnetohydrodynamic configurations that these magnetic fields exert a negative pressure and tend to accelerate the expansion of the Universe.

The reason that we know so accurately the value of the CC is that, fortunately, the combination of the nonrelativistic mass density Ω_M and CC density Ω_Λ as measured using supernovae is almost orthogonal to the combination probed by the cosmic microwave background anisotropy spectrum. It is then possible to estimate the values of these two quantities with precision[5,6,4], and it turns out that $\Omega_M \approx 0.3$ and $\Omega_\Lambda \approx 0.7$.

The equations of general relativity for a *spatially flat* Robertson-Walker (homogeneous and isotropic) universe are[7] the expansion rate equation

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda \tag{1}$$

and the acceleration of the cosmological expansion equation

$$3\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p) + \Lambda. \tag{2}$$

(We are using the 'natural units' common in quantum field theory, that take $c = \hbar = 1$.) Here a is the cosmic expansion factor, ρ is the energy density, p the pressure and Λ the CC. Notice that with null pressure and density the equations have a solution of the form $a = e^{t/\tau}$, that is, an inflationary solution. There has been one known mechanism to mimic the effect of a CC using a boson scalar field (sometimes called quintessence). A case in question is inflation, that uses the inflaton to simulate a CC for a short while and produce an exponential expansion even if there is no actual CC. The idea is that ρ and p should initially be fairly time-independent and satisfy $p \approx -\rho$. This situation can be achieved using the inflaton, whose Lagrangian density of a scalar field would have the form $\mathcal{L}_\varphi = \frac{1}{2} g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi)$. The first term is the kinetic energy and the second minus the potential energy density. Since the stress tensor of a system with Lagrangian $\mathcal{L}$ that does not depend on the metric's derivatives (the usual case) is given by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g^{\mu\nu}}, \tag{3}$$

we obtain for the density and pressure of this field $\rho_\varphi = T_{00} = \frac{1}{2}\dot{\varphi}^2 + V$ and $p_\varphi = T_{ii} = \frac{1}{2}\dot{\varphi}^2 - V$, where we have used Eq. (3) to find the time component T_{00} and one of the space components T_{ii}, $i = 1, 2, 3$. We are also neglecting the spatial gradients in the formulae for the density and the pressure. If the potential $V(\varphi)$ is fairly flat for the initial value that φ takes and if $\dot{\varphi}$ is small initially, then the field is going to *slowly* move along the potential V. The small kinetic leads one to conclude that $\rho_\varphi \approx V \approx -p_\varphi$ for a while. Using this relation in Eq. (2) we can verify right away the right-hand side of the equation is positive, even for $\Lambda = 0$. The acceleration of the expansion of the Universe will temporarily have a positive sign.

We proceed to show, using similar ideas, that the energy density ρ_B and pressure p_B due to cosmic magnetic fields also obey $\rho_B \approx -p_B$, so that cosmic magnetic fields contribute as a positive CC to the Universe's acceleration. The Lagrangian of the electromagnetic field is given by $\mathcal{L}_B = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, and from it we can find the density

$$\rho_B = T_{00} = \frac{1}{2}(E^2 + B^2) \tag{4}$$

and

$$p_B = T_{ii} = \frac{1}{2}(E^2 - B^2). \tag{5}$$

We will show in a moment that $E^2 \ll B^2$; assuming this for the time being notice $\rho_B \approx -p_B$, so that Eqs. (1) and (2) take the form

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho + \Lambda_B \tag{6}$$

and

$$3\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p) + \Lambda_B. \tag{7}$$

In these equations we have set $\Lambda = 0$ and defined $\Lambda_B \equiv 4\pi G B^2$, and the quantities ρ and p by definition do not to include the contributions due to cosmic electromagnetic fields. The implication of Eqs. (6) and (7) is that the cosmic magnetic field mimics a positive CC.

We now show that it is actually true that $E^2 \ll B^2$. Even though there are a few high-speed charged particles shooting through the Cosmos, most form a nonrelativistic, neutral plasma that is assumed to be an excellent conductor. In fact, assuming the plasma to be infinitely conducting, we have Ohm's law for plasmas[8]

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0. \tag{8}$$

Since $\mathbf{v}$, the particle's velocity in units of the speed of light, is very small, just a few meters per second, it can be concluded that in the interstellar plasma electric fields are much weaker than the magnetic ones, and that therefore cosmic magnetic fields contribute to the acceleration of the expansion of the Universe as a positive CC.

What we have said about cosmic magnetic fields bears no relation whatsoever to the magnetic fields of photons. While cosmic magnetic fields are large coherent structures that slowly drift along with the plasma, photons are small incoherent excitations travelling at the speed of light, and it goes without saying that a photon gas always exert a positive pressure. The quantization of photons, in the radiation gauge, for example, requires the absence of currents and charges. This is a very interesting situation, very similar to the one of the inflaton. An *incoherent* scalar particle gas tends *to slow* the acceleration of the Universe, while the macroscopically *coherent* field φ, that is the solution to the Euler-Lagrange equations generated by the Lagrangian density $\mathcal{L}_\varphi$ under the initial conditions we mentioned above, tends *to accelerate* this expansion.

While the cosmic magnetic fields act as a positive CC, they do not seem to be the cause of the acceleration we are observing nowadays. They are too weak. Let us estimate the magnitude of the magnetic field necessary for the observed cosmological acceleration. From the information contained in the references already cited we conclude that, approximately, $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ and the curvature is zero. We switch to Gaussian units to obtain the magnetic field in Gauss. The critical mass density is $\rho_c = 3H_0^2(8\pi G)^{-1} = 1.9 \times 10^{-29} h_0^2$ g-cm^{-3}, so that taking $h_0 = 0.7$ the energy density due to the CC normalized density is $u = \rho_c c^2 \Omega_\Lambda = 5.9 \times 10^{-9}$ erg-cm^{-3}. If this density were due to magnetic fields then $u = B^2/8\pi$, and $B = 380\mu G$. The macroscopic magnetic fields that have been detected in galaxies, galactic halos and clusters[9,10] have values of a few μG. Assuming an average value of $10\mu G$ they are too weak by a factor of 38 times to be the source of the CC. The fields of the interstellar voids are believed to be several orders of magnitude smaller. So, unless we have been grossly underestimating the intensity of cosmic magnetic fields, an unlikely situation, they are far too weak to be the cause of the observed CC.

A couple of comments are in order with respect to the effect we have been discussing in this paper, of having cosmic magnetic fields acting as a CC. The first is that it is the cooling of the plasma what brings about the effect, as this is what make the electric field so much weaker than the magnetic. It should be interesting to review the history of the Universe with this in mind. The second is that this effect can occur in other Yang-Mills theories, because these theories can be written in in terms of electric-like and magnetic-like fields[11], with T_{00} and T_{ii} looking similar to Eqs. (4) and (5) for electromagnetism, but summed over the non-abelian quantum numbers. If dark matter were made up of Yang-Mills fields then this is a possibility to consider, as dark matter could act as a CC.

References

1. Garnavich, P. M. et al. Constraints on cosmological models from Hubble Space Telescope observations of high-z supernovae. *Astrophys. J.* **493,** L53-L57 (1998).
2. Perlmutter, S. *et al.* Discovery of a supernova explosion at half the age of the universe and its cosmological implications. *Nature* **391**, 51-54 (1998).
3. Schmidt, B. P. *et al.* The high-z supernova search: measuring cosmic deceleration and global curvature of the Universe using type Ia supernovae. *Astrophys. J.* **507**, 46-63 (1998).
4. Perlmutter, S. *et al.* Measurements of Ω and Λ from 42 high-redshift supernovae. To be published in the *Astrophys. J.*
5. White, M. Complementary Measures of the mass density and cosmological constant. *Astrophys. J.* **506,** 495-501 (1998).
6. Eisenstein, D. J., Hu, W. & Tegmark, M. H_0 and Ω_m from Combining Cosmic Microwave Experiments and Redshift Surveys. *Astrophys. J.* **504,** L57-L60 (1998).
7. See, for example, Peebles, P. J. E. *Principles of Physical Cosmology* (Princeton University Press, 1993).
8. Biskamp, D. *Nonlinear Magnetohydrodynamics* (Cambridge University Press, 1997).
9. Enqvist, K. Primordial magnetic fields, Los Alamos astro-ph/9803196. This paper reviews material on cosmic fields and possible mechanisms for their generation. Also see Chaves, M. & Padilla, G. Los Alamos gr-qc/9810018.
10. Olinto, A. V. Cosmological magnetic fields. *Proc. of the 3rd RESCEU Symposium, Univ. of Tokyo, Nov. 10-13, 1997,* (Ed. K. Sato, T. Yanagida and T. Shiromizu).
11. Huang, K. *Quarks, Leptons and Gauge Fields, 2 ed.* (World Scientific, 1998).